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Modeling the Growth of Jumpers on the Main Distributing Frame

By S. HALFIN, C. J. McCALLUM, JR., and M. SEGAL

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The buildup of dead jumpers in the Main Distributing Frame (MDF) plays a central role in MDF problems; for example, a recent survey ranked dead jumpers as the number two problem on a list of the most frequently reported MDF problems. In this paper, two models are proposed to quantify the buildup of both live and dead jumpers and to investigate the factors influencing the buildup. These models provide tools for the analysis and comparison of possible solutions to the buildup problem.

I. INTRODUCTION

The Main Distributing Frame (MDF) in a central office building serves as the connecting point between the cable outside and the equipment inside. Conventional MDF's are iron or wooden structures with terminal strips mounted on each side. The two sides are termed *vertical* and *horizontal* due to the manner in which the terminal strips are mounted. Cable pairs from subscribers' stations are terminated on the vertical side of the MDF, while line and trunk equipments are wired to terminals on the horizontal side of the frame. In order to provide service to a subscriber, it is necessary to connect his cable pair to the proper line equipment. A frameman makes this cross-connection by manually stringing a wire, called a *jumper*, between the corresponding vertical and horizontal terminals. These jumpers, which can easily be 100 feet long, are laid along horizontal shelves in the frame. As more and more jumpers are added to the frame, these horizontal shelves tend to become crowded.

When the service to a particular station is discontinued, it is necessary to manually disconnect and remove the corresponding jumper. Unfortunately, disconnected jumpers are not always removed, which gives rise to *dead* jumpers in addition to the *live* jumpers.

In the last few years, main frames which were originally conceived

over seventy years ago are becoming highly congested with live, and dead, jumpers.[1] The problem of the buildup of jumpers on the shelves of the MDF has been discussed extensively and many solutions have been suggested and examined.[2,3] These proposals include preferential assignment of line equipments to cable pairs, multipling line equipments to several appearances on the horizontal side of the frame, spreading line equipments along the horizontal side, using several separated frames connected by tie cables, mechanization of the assignment records, and others. A natural way to evaluate such proposals is by means of a growth model for the buildup of jumpers in the MDF. Desirable features of such models are:

(*i*) The models should enable the comparison of the amount of live and dead jumpers during a given period of time, for various conditions and methods of administration of the MDF.

(*ii*) The models should use as inputs either data which are easily available or which can be theoretically deduced from available data. Such data are: the physical description of the frame, the rate of connect and disconnect orders, distribution of lengths of the jumpers, which can be deduced from administrative practices (for instance, the method of assignment), etc.

One would expect such models to be able to clarify why some MDF's suffer from dead jumper accumulation, while others, though apparently similar, do not. Also, the models should facilitate evaluation of the proposed solutions.

In this paper, we investigate the jumper buildup in a frame over time, with emphasis on the dead jumper buildup. Two models are proposed to quantify this buildup.

The first model is based on the assumption that the fraction of dead jumpers not removed from the frame has a certain functional form which depends on two presumably measurable parameters which are described later. Given values of these two parameters and the values of two other frame parameters which are easily determined, the model can be used to iteratively calculate the jumper buildup over time starting from some initial state.

In the second model, it is assumed that a dead jumper remains on the shelf either because its assignment record is erroneous, or because the force needed to remove it is too large. Based on these assumptions, recursive equations are deduced for the quantity of jumpers on a shelf at any given time.

In Section II, the first model is developed and some numerical results

are given. Section III is concerned with the second model. Some concluding comments are given in Section IV.

II. DEVELOPMENT OF MODEL 1

2.1 *The Model*

Let $t = 0, 1, \cdots$ denote discrete points in time. Time period t will refer to the time period between time t and time $t + 1$. Let $J(t)$ be the total number of jumpers (both live and dead) in the frame at time t; equivalently, $J(t)$ is the number of jumpers in the frame at the beginning of time period t before any service orders for that time period are processed. Let $c(t)$ denote the number of connects (i.e., service orders which require the installation of a jumper) that occur during time period t. Similarly, let $d(t)$ denote the number of disconnects (i.e., service orders which require the disconnection and removal of a jumper) that occur during time period t. For ease in exposition, we assume that each service order calls for the connection or the disconnection of only one jumper.

Using the terms introduced above, we can write $J(t + 1)$ as

$$J(t + 1) = J(t) + c(t) - \beta(t)\, d(t), \qquad t = 0, 1, \cdots \tag{1}$$

where $\beta(t)$, defined as the fraction of the disconnects $d(t)$ that are removed, accounts for the fact that disconnected jumpers are sometimes left in the frame. Clearly, $0 \leqq \beta(t) \leqq 1$.

It is convenient to introduce the substitution $\beta(t) = 1 - \alpha(t)$ into eq. (1) where $\alpha(t)$ represents the fraction of disconnects that are *not* removed. Note that $0 \leqq \alpha(t) \leqq 1$. This gives

$$J(t + 1) = J(t) + c(t) - d(t) + \alpha(t)\, d(t) \tag{2}$$

where $c(t) - d(t)$ represents the net gain in live jumpers during time period t. Similarly, the expression $\alpha(t)\, d(t)$ represents the increase in dead jumpers during time period t. It is clear that we favor the situation in which $\alpha(t)$ is near 0 (equivalently, $\beta(t)$ is near 1). In such a case, the jumper buildup in time will correspond only to live jumpers added to the frame and the additional frame congestion brought on by the presence of dead jumpers will not be felt.

We now simplify the model and remove the dependence of $c(t)$ and $d(t)$ on time. Let S be the number of service orders per time period and let G be the fraction of the service orders that result in net gain, i.e.,

$$S = c(t) + d(t) \tag{3}$$

$$G = \frac{c(t) - d(t)}{c(t) + d(t)} \tag{4}$$

for all $t = 0, 1, \cdots$. We assume $S > 0$. Note that in the usual case in which $c(t) \geqq d(t)$ we have $0 \leqq G \leqq 1$. Using eq. (3) and eq. (4) to eliminate $c(t)$ and $d(t)$ from eq. (2), we have

$$J(t + 1) = J(t) + GS + \tfrac{1}{2}(1 - G)S\alpha(t). \tag{5}$$

2.2 *The Parameter* $\alpha(t)$

We now investigate the parameter $\alpha(t)$, the fraction disconnects not removed. Though S and G were taken to be time independent, this seems to be an unreasonable assumption to make for $\alpha(t)$. One might expect that $\alpha(t)$ will increase with time as the number of jumpers in the frame increases. That is, when there are few jumpers in the frame (i.e., $J(t)$ is small) we expect that most disconnects will actually be removed (i.e., $\alpha(t)$ is near 0) since the frame is certainly not congested and only the disinclination of a frameman to remove an occasional disconnect or an error in the records will lead to its being left in the frame. However, as $J(t)$ increases, the frame becomes congested due to jumper buildup, and we expect that, due to physical limitations, fewer and fewer of the disconnects will actually be removed (i.e., $\alpha(t)$ is near 1). The removal of a disconnect will become physically difficult if not impossible and could jeopardize the continued operation of nearby live jumpers. Thus, we define $\alpha(t)$ to be an increasing function of $J(t)$ with a range of 0 to 1. Furthermore, we require that $\alpha(t)$ actually reaches the value 1, i.e., at some time t^* when $J(t^*) = K$, we have $\alpha(t^*) = 1$. K, the jumper removal congestion number, indicates the number of jumpers at which disconnects can no longer be removed. In practice, frames reaching this stage require complete replacement or extensive cleanup campaigns. Note that when disconnects are not being removed, the growth rate of $J(t)$ is the full rate of the connects. The assumed form of $\alpha(t)$ is

$$\alpha(t) = \begin{cases} \left[\dfrac{J(t)}{K}\right]^{\gamma}, & J(t) \leqq K \\ 1, & J(t) > K \end{cases} \tag{6}$$

where γ, a non-negative real number, indicates the rate at which $\alpha(t)$ approaches 1. To see the effect of the number γ on $\alpha(t)$, refer to Fig. 1. For $\gamma = 1$, $\alpha(t)$ increases linearly with $J(t)$. For $\gamma = 2$, $\alpha(t)$ starts more slowly with respect to $J(t)$ but then accelerates. The extreme case in

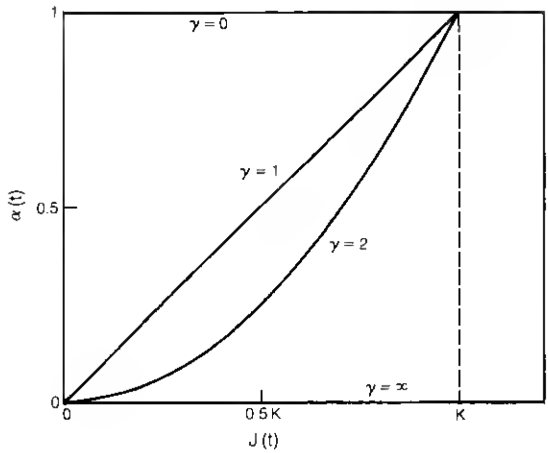


Fig. 1—Graph of $\alpha(t)$ vs $J(t)$.

which $\gamma = \infty$ corresponds to the case in which $\alpha(t) = 0$ for $0 \leqq t \leqq t^*$; this is the situation in which all disconnects are removed until the live jumper buildup exceeds K jumpers. For $\gamma = 0$, we have the extreme case in which $\alpha(t) = 1$ for all t, i.e., no disconnects are ever removed.

A number of factors influence the parameters K and γ. Those factors determining K are primarily frame parameters determined by the hardware of the frame and its use. One such factor is the frame configuration which includes such items as the number of verticals on the frame, the number of shelves, the positioning of various terminal densities on the vertical and horizontal sides of the frame, and the presence of bends in the frame (if any). Another factor influencing K is the jumper assignment procedure which, in part, determines neatness of the frame. For example, one might intuitively expect that preferential assignment would smooth out the jumper congestion inherent in random assignment. Thus, a frame would be able to accommodate a great number of jumpers before dead jumpers could no longer be removed. Hence, K would be larger under preferential assignment.

The factors determining γ are generally related to the human environment. One such factor is record quality. A frameman who attempts to remove a disconnect based on instructions derived from erroneous records may not be able to complete the task. In such a situation, the jumper in question will usually become a dead jumper. Other factors influencing γ are the numbers of both framemen and frame foremen and their skills and motivation.

Note that the model assumes that all hardware features and administrative procedures of a frame are tied together in the parameters K and γ. This feature of the model allows us to treat radically different configurations, assignment procedures, etc. This abstraction is in contrast to the work of Swanson[4] in which physical properties of the frame are considered explicitly.

If we substitute the form of $\alpha(t)$ given in eq. (6) into eq. (5), we get

$$J(t+1) = \begin{cases} J(t) + GS + \frac{1}{2}(1-G)S\left[\dfrac{J(t)}{K}\right]^{\gamma}, & t \leqq t^* \\ J(t) + \frac{1}{2}(1+G)S, & t > t^*. \end{cases} \tag{7}$$

Note that as t passes t^*, $\alpha(t)$ becomes 1, and the future growth in $J(t)$ is independent of K and γ. Given the parameters G, S, K, and γ, eq. (7) allows us to iteratively calculate the jumper buildup over time starting from some initial value $J(0)$. In numerical calculations, $J(0)$ is usually taken to be 0 though it would also be reasonable to interpret $J(0)$ as the number of jumpers in the frame at its cutover time, which might be a substantial positive quantity. Note that if $J(0)$ is not equal to 0, then $\alpha(0)$ is a number greater than 0. If one desires to grow $\alpha(t)$ from 0, he may replace $\alpha(t)$ in eq. (6) (and then eq. (7)) by

$$\alpha^*(t) = \begin{cases} \left[\dfrac{J(t) - J(0)}{K - J(0)}\right]^{\gamma}, & J(t) \leqq K \\ 1, & J(t) > K. \end{cases} \tag{8}$$

2.3 *Computer Program and Numerical Results*

A computer program was written to perform the iterative calculations of eq. (7). It is worth noting that for cases in which γ is a non-negative integer, it is unnecessary to perform the iterative scheme of eq. (7) since $J(t)$ can be obtained directly by solving a nonlinear first order differential equation.

The jumper buildup $J(t)$ is now examined for typical values of the parameters S, G, γ, and K. Let $S = 300$ service orders per day and let the gain fraction $G = 0.05$. Assume $\gamma = 2$ which seems to be reasonable. Typical estimates for reasonable values of K tend to be in the range of fifty to one hundred percent greater than the terminal capacity of the frame. By terminal capacity, denoted C, we mean that design parameter of the frame which indicates the maximum number of *live* jumpers that the frame can accommodate. For example, C might equal the number of horizontal terminals of the frame. We shall take $C = 80{,}000$ jumpers and $K = 120{,}000$ jumpers. Finally, let $J(0) = 0$.

The jumper buildup based on the above parameter values is graphed versus time (in days) in Fig. 2. The straight line in the graph indicates the buildup of live jumpers alone due to the gain GS. At the rate of $GS = 15$ jumpers per day, the terminal capacity C is exhausted after 5334 days or approximately 20 years considering 270 working days per year. At this point of 5334 days, $J(t)$ equals 446,033 jumpers with only 18 percent of these jumpers live. The point in the graph surrounded by a small box indicates the time $t^* = 3264$. At this time, $\alpha(t)$ has become equal to 1, and $J(t)$ is growing linearly at the rate of the connects $\frac{1}{2}(1 + G)S = 157.5$ jumpers per day or 10.5 times the rate of the gain.

Figure 3 considers various values of K for the fixed value of $\gamma = 2$. In this graph, $J(t)$ is plotted on a logarithmic scale. Notice that $J(t)$ decreases, for fixed values of t, as we increase K. The line corresponding to $K = \infty$ represents the case in which the jumper buildup is due to live jumpers alone through the gain GS.

In Fig. 4, we consider the case of $K = 120{,}000$ with various values of γ. Note that $J(t)$ decreases, for fixed values of t, as γ is increased. Note also that t^* is an increasing function of γ. We see that by controlling the factors that influence γ so as to increase γ, we postpone the time t^* at which point we no longer remove dead jumpers. The case of $\gamma = \infty$

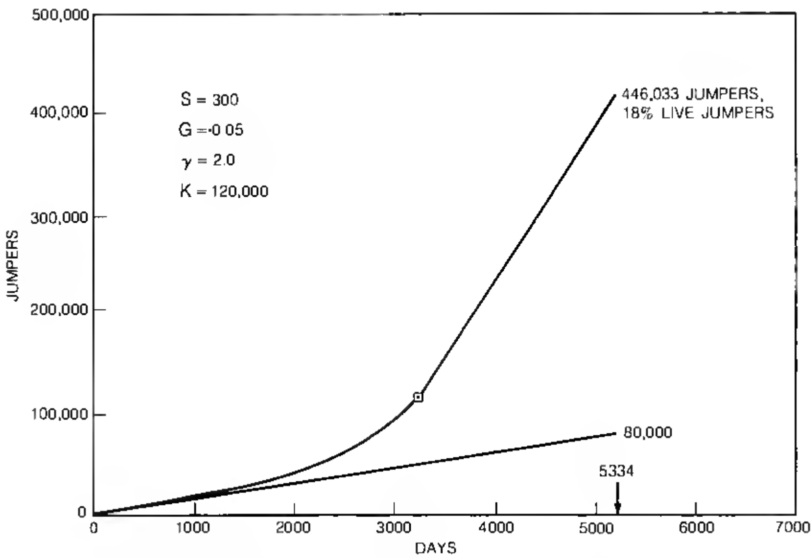


Fig. 2—Jumpers vs time for $\alpha = 2$ and $K = 120{,}000$.

corresponds to the ideal situation in which the jumper buildup is due only to live jumpers.

It should be stressed that the functions $J(t)$ given in Figs. 2, 3, and 4 follow from our assumed choice of parameter values. To actually use the model and predict jumper growth in a frame, one must estimate parameter values for the particular frame in question.

2.4 *Use of the Model*

One suggested use of the model is to predict the "breathing time" that a frame has before total congestion sets in. Presumably there is some total congestion number (in jumpers) such that when $J(t)$ reaches this number, the frame can no longer operate but must shut down. This total congestion number should not be confused with K, the jumper removal congestion number. Given the values of the parameters and the current jumper buildup, the model can be used to predict the time at which $J(t)$ equals the total congestion number. The time until this total congestion time is the "breathing time."

Any use of the model is predicated on our ability to make good estimates of the values of the model parameters S, G, K, and γ. Data is readily available for the parameters S and G. However, K and γ are not so easy to estimate. Though more work needs to be done on ways

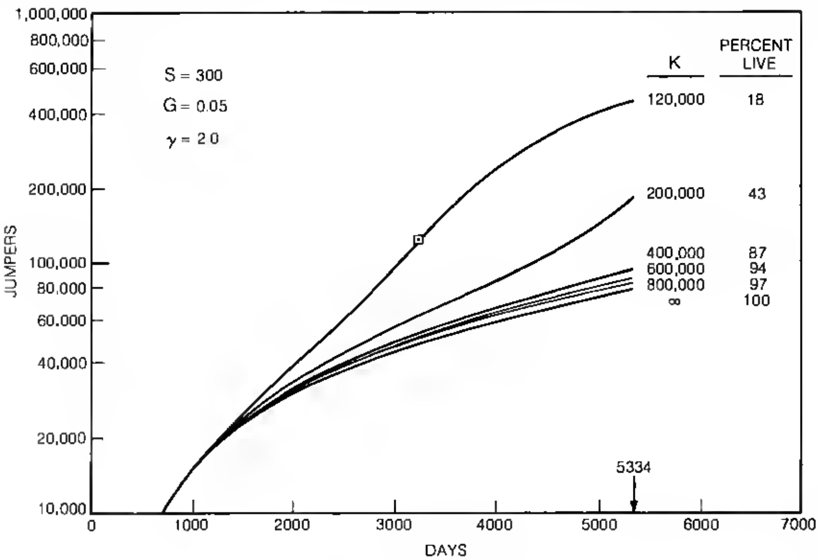


Fig. 3—Jumpers vs time for $\gamma = 2$.

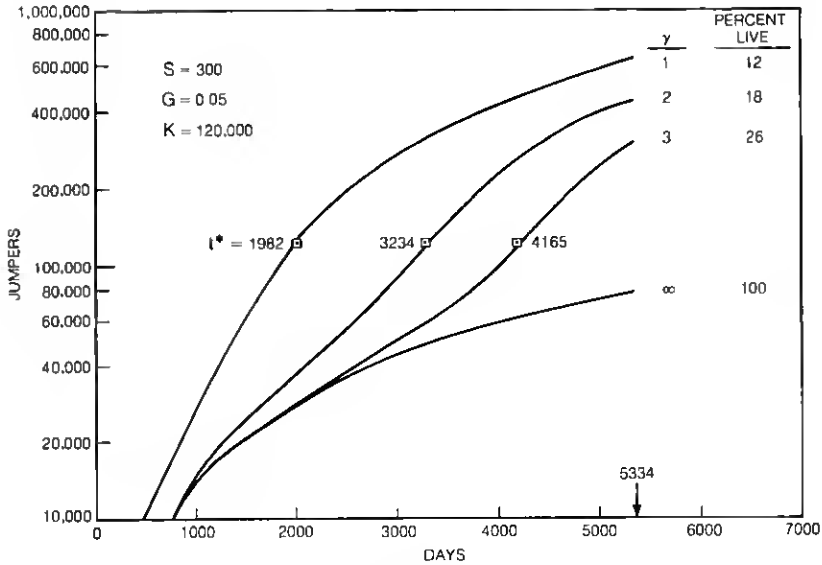


Fig. 4—Jumpers vs time for $K = 120{,}000$.

to properly estimate K and γ, it is sufficient to obtain an estimate of $J(t)$ for two or more (nonzero) values of t to determine these parameters.

Numerous refinements of the model can be considered. One could take S and G as functions of time as they most certainly are. In addition, although the form of $\alpha(t)$ given in eq. (6) is a reasonable starting point, more realistic functions need to be considered. One such effort in this direction is the model developed in the next section. That model which is built on a slightly different approach introduces parameters that are easier to estimate. Both friction and the weight of jumpers in the frame are explicitly considered as well as the percentage of errors in the assignment records.

III. DEVELOPMENT OF MODEL 2

3.1 *The Model*

We make the following assumptions:

(*i*) The frameman removes a disconnected jumper if the following two conditions are satisfied: there is no error in the assignment record of the jumper, and the force needed to pull it out is not greater than R_0 pounds.

(*ii*) The force required to pull out a jumper is proportional to its length, and to the total weight of all the jumpers which were put in the shelf after it.

(*iii*) The length of a jumper, the correctness of its record, and whether or not it will be disconnected at a given time are statistically independent events.

Let us denote by X_t the length of a live jumper which was installed at time t, let $F(x, t) = Pr(X_t \leqq x)$ be the distribution function of X_t, and let $\bar{X}_t$ denote the expected value of X_t. Let $B(t, \tau)$ be the expected total *length* of *all* jumpers at time t, which were installed in the interval $[t - \tau, t]$. Let $A(t, \tau)$ be the expected *number* of *live* jumpers at time t which were installed in the interval $[t - \tau, t]$.

Next, let $R(X, t, \tau)$ be the force required to pull out a jumper of length X at time t, given that it is of age τ. According to assumption (*ii*),

$$R(X, t, \tau) = R^*(t, \tau)X$$

where $R^*(t, \tau)$ is the force required per unit of length. $R^*(t, \tau)$ is a random variable proportional to the total weight, and hence to the total length, of all the jumpers which are of age $\leqq \tau$.

To simplify matters, we make $R^*(t, \tau)$ deterministic by replacing it by its expected value $aB(t, \tau)$ where a is a constant which depends on the weight per unit of length of a jumper, the dimensions of the shelf and the average friction of the jumpers (see Section 3.2). Note that the friction may be higher than that attributable to the theoretical friction coefficient of the jumper's coating, because the jumpers consist of twisted pairs and do not generally lie in straight lines. Thus we assume:

$$R(X, t, \tau) = aB(t, \tau)X. \tag{9}$$

Let

$$\hat{X}(t, \tau) = \frac{R_0}{aB(t, \tau)}. \tag{10}$$

Thus $\hat{X}(t, \tau)$ is the maximal length of a "pullable" jumper of age τ.

Let ϕ denote the probability of an incorrect record, and let $E(t, \tau)$ denote the average removed length of a disconnected jumper of age τ at time t. $E(t, \tau)$ is, according to assumption (*iii*), the product of the expected proportion of such live jumpers having correct records and the expected length of such live jumpers, given that their length is bounded by $\hat{X}(t, \tau)$. Hence,

$$E(t, \tau) = (1 - \phi) \int_0^{\hat{X}(t,\tau)} x d_x F(x, t - \tau). \tag{11}$$

We now introduce assumptions about the rates of connect and disconnect orders. Let us denote the connect order rate at time t by $\lambda(t)$. $\lambda(t)$ may depend on $A(t, t)$–the total expected number of live jumpers on the shelf, which represents the size of the main frame, or on $A(t, t_0)$ (where say $t_0 = 1$ year)–the recent growth of the shelf, or on external parameters such as changes in the community which is served by the central office.

Let $\mu(t, \tau)$ be the percent of disconnect orders for jumpers of age τ at time t. This rate will be, in many cases, independent of t. If a jumper is of age τ at time t, then the probability of a disconnect order for it in a small interval $[t, t + \Delta t]$ is approximately $\mu(t, \tau)\ \Delta t$.

Now let Δt be a time increment small enough so that quadratic and higher powers of Δt can be ignored. We then obtain the following system of equations:

$$\begin{aligned} B(t + \Delta t, \tau + \Delta t) &= B(t, \tau) - \Delta t \sum_{i=1}^{[\tau/\Delta t]} E(t, i\Delta t)\mu(t, i\Delta t) \\ &\quad \cdot [A(t, i\Delta t) - A(t, (i-1)\ \Delta t)] + \lambda(t)\ \Delta t\ \bar{X}_t \\ A(t + \Delta t, \tau + \Delta t) &= A(t, \tau) - \Delta t \sum_{i=1}^{[\tau/\Delta t]} \mu(t, i\Delta t) \\ &\quad \cdot [A(t, i\Delta t) - A(t, (i-1)\ \Delta t)] + \lambda(t)\ \Delta t \end{aligned} \tag{12}$$

for all t, τ such that $t \geqq \tau \geqq 0$, with the initial conditions:

$$\left.\begin{aligned} B(t, 0) &= 0 \\ A(t, 0) &= 0 \end{aligned}\right\}, \ t > 0$$

$$B(0, 0) = A_0 \bar{X}_0$$

$$A(0, 0) = A_0$$

where A_0 is the initial number of jumpers placed on the shelf.

By letting $\Delta t \to 0$, one obtains a system of partial differential-integral equations:

$$\begin{aligned} B_t(t, \tau) + B_\tau(t, \tau) &= -\int_0^\tau E(t, u)\mu(t, u)\ d_u A(t, u) + \lambda(t)\bar{X}_t \\ A_t(t, \tau) + A_\tau(t, \tau) &= -\int_0^\tau \mu(t, u)\ d_u A(t, u) + \lambda(t). \end{aligned} \tag{13}$$

Since $E(t, \tau)$ is nonlinear in the unknown function $B(t, \tau)$, it seems that an analytic treatment of these equations will not lead us far. On

the other hand, a numerical solution is readily obtained by using the recursive equation given in eq. (12).

3.2 *An Explicit Formula for a*

The constant a can be evaluated in the following way. Denote

l—the length of the shelf.
w—the width of the shelf.
σ—the weight of a pair of jumpers per unit of length.
ρ—the number of pairs of jumpers in a unit of area in the cross-section of the shelf.
δ—the friction coefficient of the pairs of jumpers. (δ is the force needed to pull a pair per unit of vertical force which is applied on the pair.)

We assume that the buildup of jumpers is homogeneous throughout the length and the width of the shelf, and that one jumper lies higher than another jumper if and only if it was installed later. Likewise we assume that there are $w\sqrt{\rho}$ pairs of jumpers in any horizontal layer at any cross section of the shelf.

Let us consider a pair of jumpers of length X which was installed at time $t - \tau$, and let the present time be t. Then the total vertical weight on the pair is

$$\sigma B(t, \tau) \frac{X}{l} \frac{1}{w\sqrt{\rho}}.$$

Thus the force which is needed to pull it out is

$$R(X, t, \tau) = \delta\sigma B(t, \tau) \frac{X}{l} \frac{1}{w\sqrt{\rho}}.$$

Comparing this formula with eq. (9) we obtain

$$a = \frac{\delta\sigma}{lw\sqrt{\rho}}.$$

3.3 *Distributions of Jumper Lengths*

One of the major factors determining the behavior of the main frame is the distribution of jumper lengths. One might expect that the shorter the jumpers, the slower is the growth of the mass of jumpers on a frame. There are two reasons for this phenomenon: (*i*) short jumpers have small mass, and (*ii*) short jumpers are easier to pull out when disconnected. The distribution of jumper lengths is determined, and hence can be controlled, by the method used for assigning vertical terminals

to horizontal terminals. Various administrative and planning methods have been proposed, such as preferential assignment, duplication of equipment terminals, etc. We believe that with the present model one can analyze the effect of any of these methods on the buildup of jumpers. To perform such an analysis one needs to determine the distributions $F(x, t)$ for the various methods. We present here the distributions for the case of random assignment, and for the case of preferential assignment with leakage.

Random assignment yields the following distribution:

$$F(x, t) = \begin{cases} \frac{1}{l^2} x(2l - x) & \text{for} \quad 0 \leqq x \leqq l \\ 1 & \text{for} \quad l < x \end{cases}$$

where l is the length of the shelf. Note that, in general, l will be a function of t due to growth of the frame.

By preferential assignment with leakage we mean that the MDF is divided into N zones with equal length l/N. We assume that there are two types of connect orders: type 1, which can be connected within the home zone, and type 2, which, for various reasons, are randomly assigned over *all* zones. Let ζ be the fraction of type 2 orders among all the orders. $0 \leqq \zeta \leqq 1$. ζ will be called the *leakage coefficient*. Our model is somewhat different from the model of Swanson.[4] There, the leakage is defined as the fraction of those orders which cannot be executed within one zone. Note that in our case, type 2 orders may still be executed in the home zone. If Swanson's leakage coefficient is denoted by ζ_s then we have

$$\zeta_s = \frac{N - 1}{N} \zeta.$$

The distribution of jumper length for preferential assignment with N zones and leakage coefficient ζ is:

$$F(x, t) = \begin{cases} (1 - \zeta) \frac{xN^2}{l^2} \left(2 \frac{l}{N} - x\right) + \zeta \frac{x}{l^2} (2l - x) & \text{for} \quad 0 \leqq x \leqq \frac{l}{N} \\ 1 - \zeta + \zeta \frac{x}{l^2} (2l - x) & \text{for} \quad \frac{l}{N} < x \leqq l \\ 1 & \text{for} \quad l < x. \end{cases}$$

As before, the length l of the frame, the number of zones and the leakage coefficient may be functions of t, that is, $l = l(t)$, $N = N(t)$ and $\zeta = \zeta(t)$. The case $\zeta = 1$ corresponds to the random assignment method.

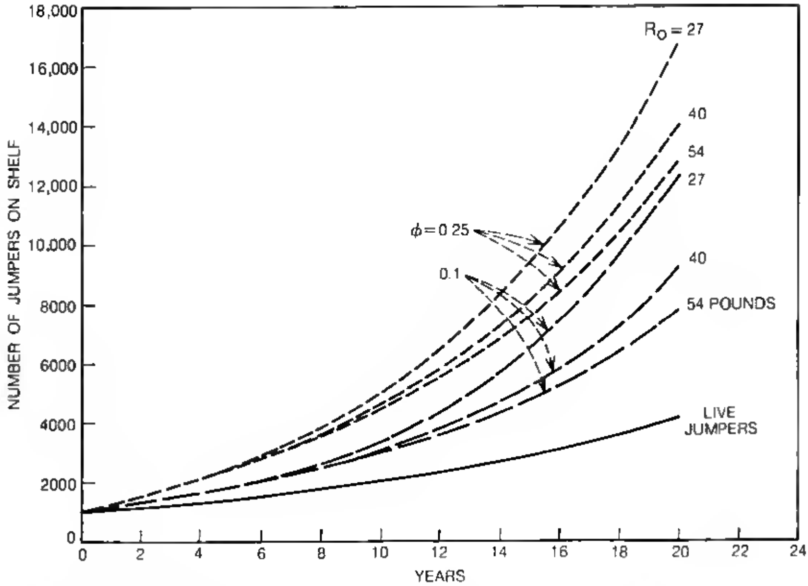


Fig. 5—Growth curves for various values of the maximal pulling force R_0, and the fraction of erroneous records ϕ, for random assignments.

3.4 *Computer Program and Numerical Results*

The model was programmed for the case of preferential assignment with leakage, where the length of the frame, the number of zones and leakage factor remain constant over time. As mentioned before, this includes random assignment as a special case. We made the following simplifying assumptions about the connect and disconnect processes:

(*i*) $\lambda(t) = \lambda_0 A(t, t)$, namely, the rate of connect orders is linear with respect to the total number of live jumpers.

(*ii*) $\mu(t, \tau) = \mu_0$, namely, the percent of disconnect orders is identical for all age groups of jumpers, and is independent of time.

These assumptions may be changed by minor modifications of the program.

The program solves the system of equations given in eq. (12) for a given time step size Δt. The user must specify $\lambda_0\ \Delta t$ and $\mu_0\ \Delta t$. Note that Δt should not be too large, otherwise eq. (12) will not be valid. A good criterion for the size of Δt is that $A(t, t)$ and $B(t, t)$ should not be much different from $A(t + \Delta t, t + \Delta t)$ and $B(t + \Delta t, t + \Delta t)$ respectively (say by 1 percent at most).

Other input parameters are: R_0, a, l, ϕ, ζ, A_0, N and finally n, the desired number of iterations, so that the equations are solved up to the time $T_{max} = n\ \Delta t$. At the starting point for the iteration, $t = 0$, the shelf contains $A_0 = A(0, 0)$ jumpers.

The output gives, at various predetermined points in time, the number of live jumpers on the shelf, the total length of the live jumpers, the total length of all the jumpers, and the percentage of live jumpers in the mass on the shelf.

Sample runs of the program were made with the following input parameters:

$$\Delta t = 1 \text{ month}$$

$$\lambda_0\ \Delta t = 0.063$$

$$\mu_0\ \Delta t \equiv 0.057$$

$$n = 240 \text{ (20 years)}$$

$$A_0 = 1000 \text{ jumpers}$$

$$l = 100 \text{ ft}$$

$$a = \frac{1}{150{,}000} \text{ lb/ft}^2.$$

One set of runs was made for the case $\zeta = 1$ (random assignments) for values of ϕ ranging between 0 and 0.25, and values of R_0 ranging between 0 to 67 lb. The results are illustrated in Figs. 5, 6, and 7.

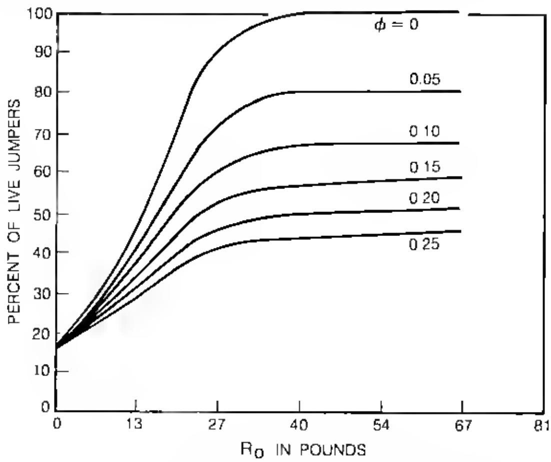


Fig. 6—Percent of live jumpers after 10 years, as a function of the maximal pulling force R_0 and the fraction of erroneous records ϕ, for random assignments.

A second set of runs was made for fixed $R_0 = 40$ lb and fixed $\phi = 0.1$ with values of ζ between 0 and 0.8 and $N = 4, 6, 8$. The results are illustrated in Fig. 8. The nominal data for this hypothetical shelf seems reasonably in line with some existing frames having approximately 7.2 percent annual growth of live jumpers; l corresponds to an MDF with 150 verticals.

The formula given previously in Section 3.2 can be used to evaluate the coefficient a. This formula requires knowledge of the friction coefficient. Since references for the magnitude of this coefficient do not appear to be readily available, we resorted to the following alternative approach.

By the same reasoning used to deduce eq. (9), we have

$$a = \frac{\tilde{R}}{\tilde{B}X}$$

where $\tilde{R}$ is the force needed to pull out the jumper, X is the length of the jumper, and $\tilde{B}$ is the total length of all the jumpers which are higher on the shelf relative to this particular jumper. Thus, a can be estimated by pulling jumpers from the shelf, noting the required force, the length, and the depth of the jumper in the pile.

Experiments of this type were conducted by Tengelsen:[5] "Readings taken thus far range from 39 pounds for a 29 foot jumper located deep in a pileup to a value of 9 pounds for a jumper approximately 100 feet long in the upper part of the pileup". We estimate that "deep in a pileup"

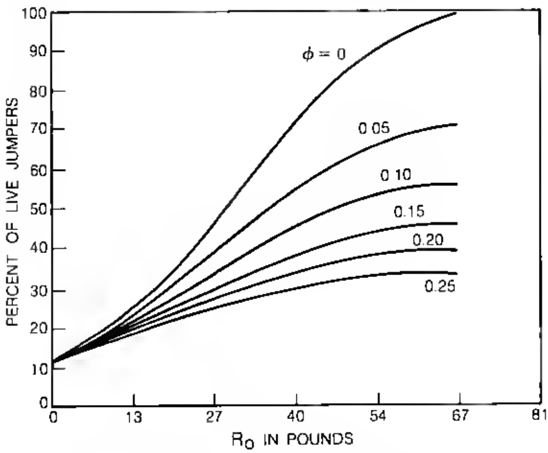
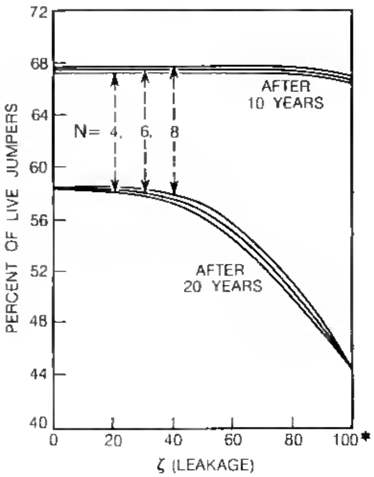


Fig. 7—Percent of live jumpers after 20 years, as a function of the maximal pulling force R_0 and the fraction of erroneous records ϕ, for random assignments.



* This corresponds to random assignment (with one zone).

Fig. 8—Percent of live jumpers as a function of the leakage coefficient and the number of zones N, for preferential assignments with $R_0 = 40$ lb and $\phi = 0.10$.

corresponds to $\tilde{B}$ of 200,000 feet. Thus

$$a = \frac{39}{29 \times 200{,}000} \sim \frac{1}{150{,}000} \text{ lb/ft}^2.$$

Finally, in the same paper (Ref. 5) it is stated that a value of 40 pounds was recommended as a safe working limit. This gave us an approximate upper limit for R_0.

From Fig. 5, one can deduce that for the range of parameters chosen, the accumulation of jumpers is more sensitive to ϕ than to R_0. This fact is also reflected in Figs. 6 and 7. In Fig. 6, we see that the percent of live jumpers on the shelf after 10 years is nearly independent of R_0 for $R_0 > 30$ lb. However, Fig. 7 indicates that after 20 years, the magnitude of R_0 is important. Figure 8 shows the influence of preferential assignment; we observe that dividing the frame into more than 4 zones seems to give no advantage when the leakage coefficient is fixed. However, one should be cautioned that, in practice, the leakage coefficient might be an increasing function of the number of zones.

IV. SUMMARY AND CONCLUSIONS

Two models for the growth of jumper buildup on the MDF have been developed. Given the values of various model parameters obtainable

from data on frame characteristics and operations, these models permit the user to predict the jumper buildup on a frame.

Many suggestions have been made for rehabilitating MDF's which are either jammed or deteriorating because of dead jumpers. Should the effort be in the direction of automating the records to reduce ϕ, or in the direction of requiring less force to remove jumpers or enabling the framemen to pull harder? Does preferential assignment offer any advantages (for a particular MDF), and if yes, into how many zones should the MDF be divided? We believe that many of these questions can be answered with the aid of the models presented here.

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